Ideal-Cipher (Ir)reducibility for Blockcipher-Based Hash Functions

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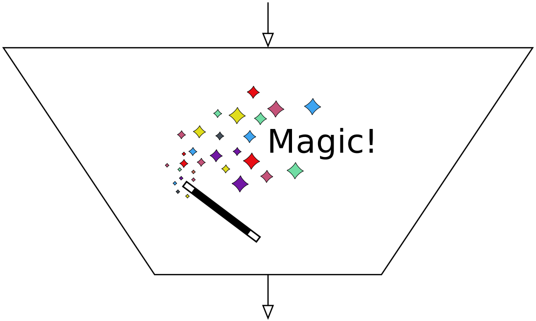
Introduction

Hash Functions in Real Life

$\{0,1\}^*$

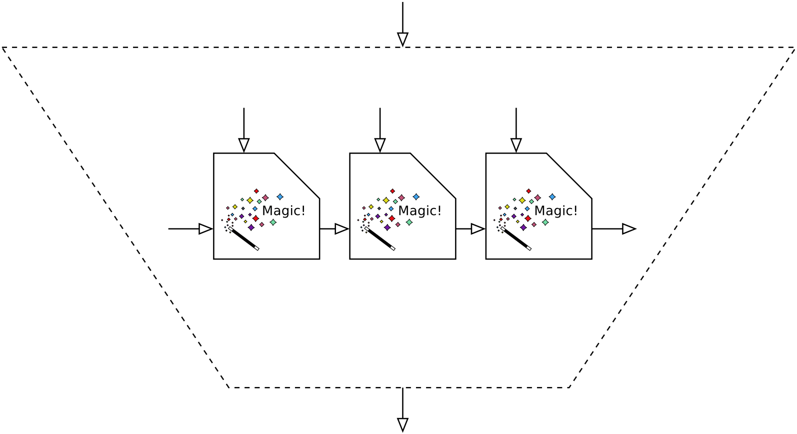


$\{0,1\}^n$

Hash Functions in Real Life

Zoom: 2x

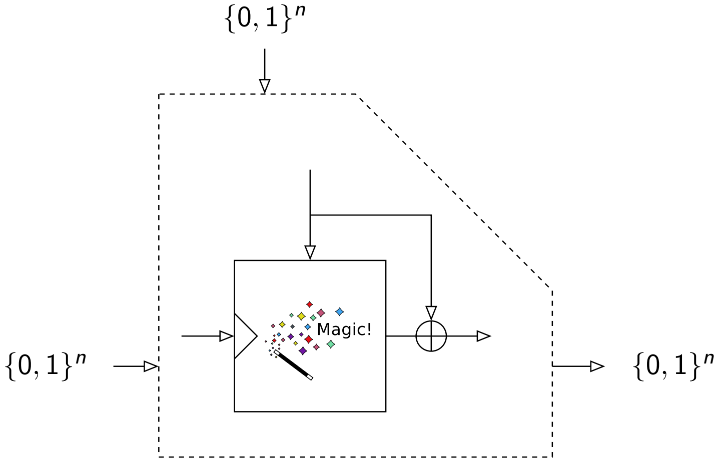
$\{0,1\}^{kn}$ for all $k \in \mathbb{N}$



$\{0,1\}^{n}$

Hash Functions in Real Life

Zoom: 3x



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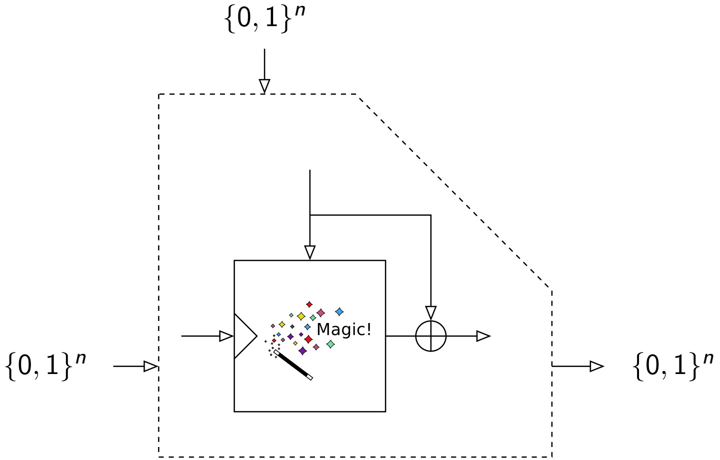
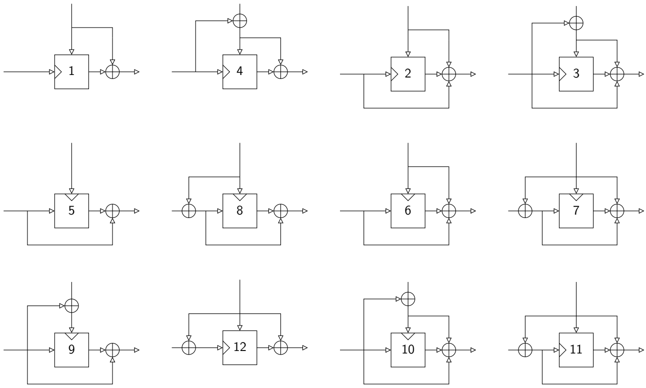


scope of this paper: blockcipher-based compression functions

Blockcipher-Based Compression Functions

- 64 basic variants using XOR operations [PGV94]
  - 12 provably secure: collision and preimage resistance [BRSS10]
  - . . . in the ideal-cipher model



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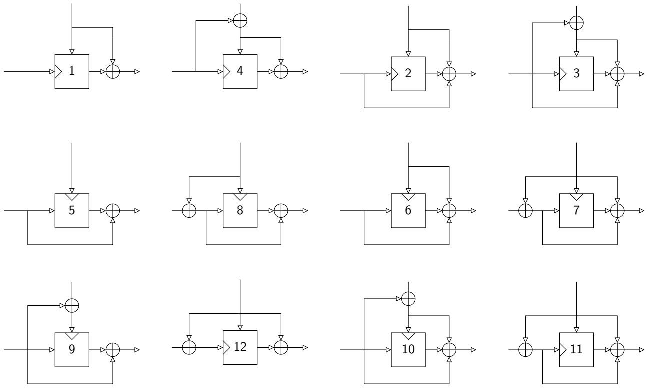

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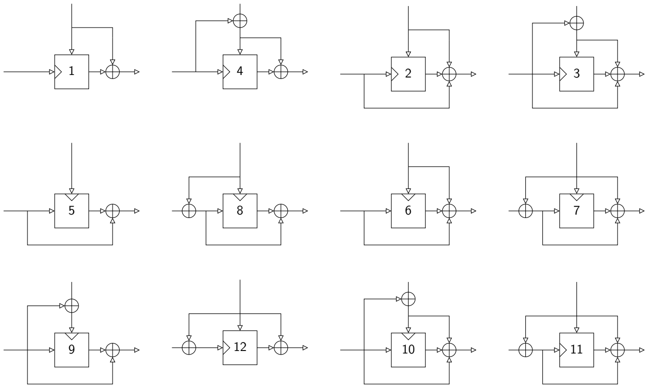

- only have AES, which function is good?

Ideal-Cipher Reducibility

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- based on (random-)oracle reducibility [BF11]

- relate compressions functions to each other w.r.t. to the blockcipher
- using a reductionist approach

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- using a reductionist approach

"any blockcipher E that makes g^{E} secure also makes f^{E} secure"

or

"the blockcipher E in f reduces to the blockcipher E in g"

Ideal-Cipher Reducibility Defined

Def.: direct reducibility

"any blockcipher E that makes g^{E} secure also makes f^{E} secure"

Def.: free reducibility

"there exists T s.t. any blockcipher E that makes g^{E} secure also makes $f^{T^{E}}$ secure"

Ideal-Cipher Reducibility Defined

Def.: direct reducibility

"any blockcipher E that makes g^{E} secure also makes f^{E} secure"

$\Longrightarrow$

T := id

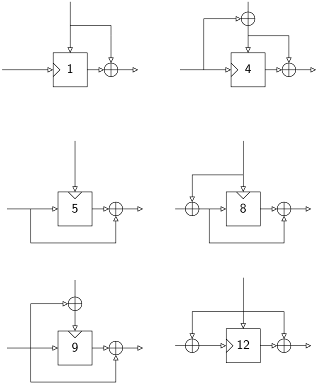
Def.: free reducibility

"there exists T s.t. any blockcipher E that makes g^{E} secure also makes $f^{\mathsf{T}^{\mathsf{E}}}$ secure"

- transformation T should be
  - simple (efficient, deterministic, stateless)
  - explicitly given in a proof
- note: simplified for exposition (E is actually a distribution)

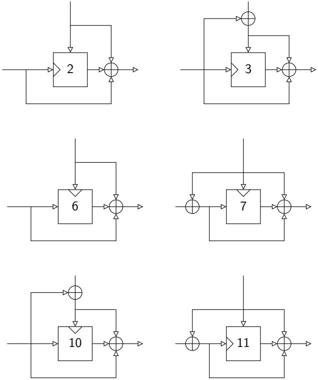
Revisiting the 12 PGV Functions

PGV_1-group



direct reducibility within

PGV_2-group



direct reducibility within

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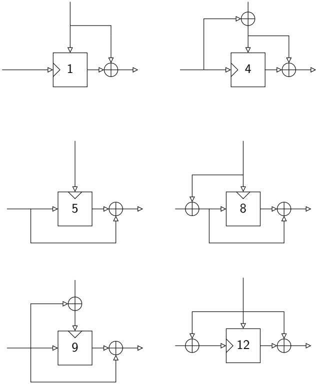
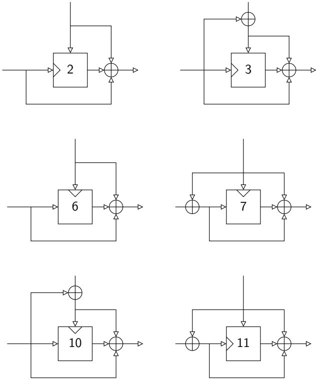


reducibility
(free)

↭

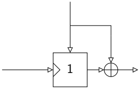
separation
(direct)

PGV_2-group



Revisiting the 12 PGV Functions

PGV$_1$-group



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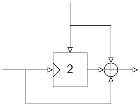
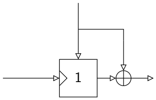
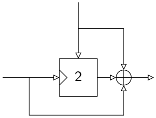


(Freely) Reducing PGV_2 to PGV_1



$\mathsf{E}(K, M) \oplus M$



$\mathsf{E}(K, M) \oplus M \oplus K$

- there exists T^{E} s.t. for any E $\quad \mathsf{PGV}_1^{\mathsf{E}}$ secure $\Rightarrow \mathsf{PGV}_2^{\mathsf{T}^{\mathsf{E}}}$ secure

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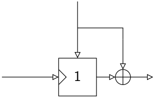

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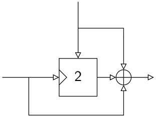
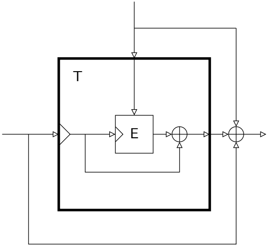


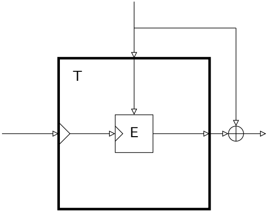
$E(K, M) \oplus M \oplus K$

- there exists T^E s.t. for any E $\quad PGV_1^E$ secure $\Rightarrow PGV_2^{T^E}$ secure
- $T^E(K, M) := E(K, M) \oplus K$



$T^E(K, M) \oplus M \oplus K$

$\equiv$



$E(K, M) \oplus M$

Revisiting the 12 PGV Functions

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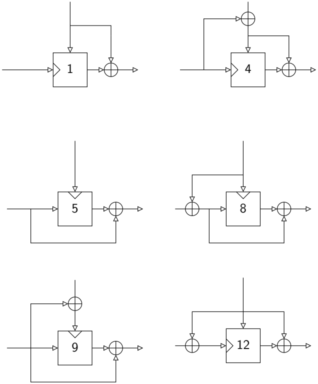
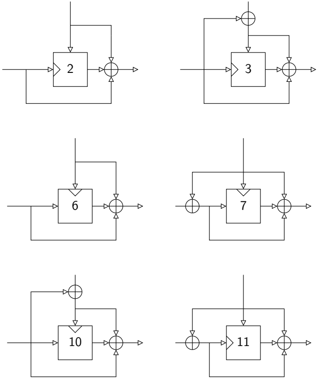


reducibility
(free)

$\leftrightsquigarrow$

separation
(direct)

PGV_2-group



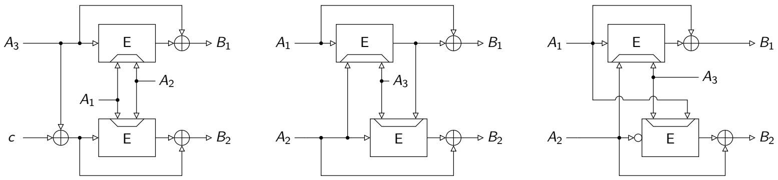
PGV Groups are Incomparable

- no direct reduction from PGV_1 to PGV_2 (or vice versa)
  - there exist blockciphers that make one secure but not the other
- groups are incomparable, no clear "winner"

Beyond PGV

Double-Block-Length (DBL) Compression Functions

- compression functions $\{0,1\}^{3n} \to \{0,1\}^{2n}$
- two blockcipher invocations, double key lengths ($2n$)



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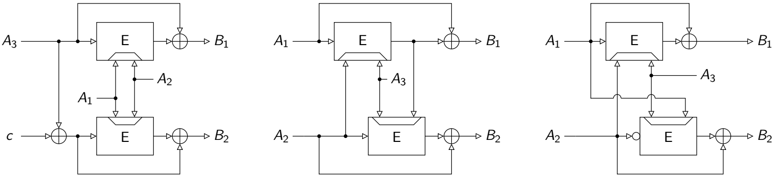

- upper part $\equiv$ PGV$_1$
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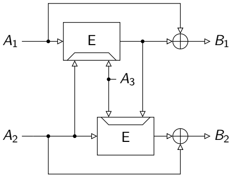

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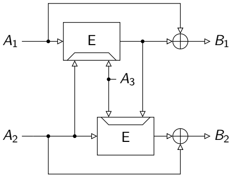

- upper part $\equiv$ PGV_1
- hence direct reducibility (only!) for collision resistance
  - preimage resistance: separation
  - idea: either output "leaks" one half of the preimage

Further Results on DBL Compression Functions

- no direct reducibility among any DBL compression function

- reducibility to PGV_1 under free transformations
  - key length extension via chaining
- no free reducibility from any PGV to any DBL
  - ... as expected?

- DBL constructions thus rely on weaker assumptions
  - i.e., not only better because of double output length

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Sketch: No Free Reducibility from PGV to DBL

- import techniques from [Pie08] on combiner impossibility
- meta reduction combined with generic bounds on attacking collision resistance [BK04]
- rule out existence of $(\mathsf{T}, \mathcal{R})$
  - $\mathcal{R}$ breaks DBL given PGV adversary:

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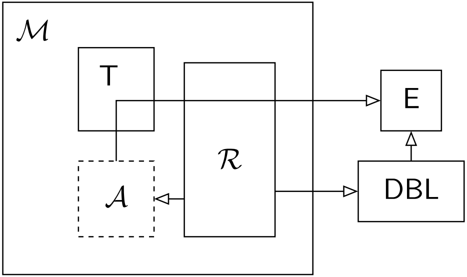

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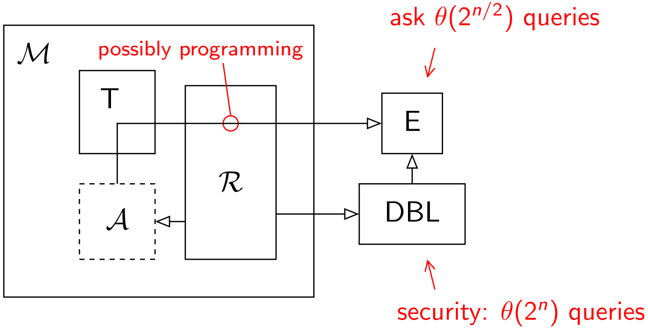

note: restrictions and fees apply

The End

Thank you!

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