Reset Indifferentiability and its Consequences

ASIACRYPT 2013

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Introduction

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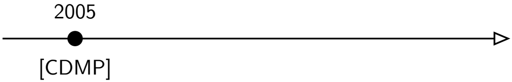

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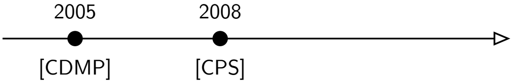

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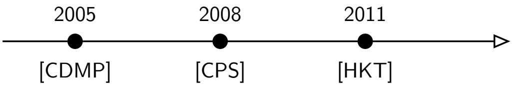

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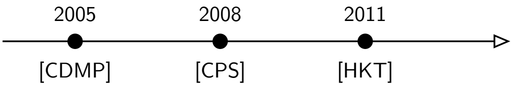

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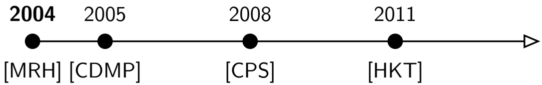
but what is "$\equiv$"?

Equivalence Through Indifferentiability



- composition theorem by Maurer, Renner, and Holenstein [MRH04]
- proof in $\color{blue}{\Pi}$ model $\rightsquigarrow$ proof in $\color{red}{\pi}$ model, given indiff. construction $\color{red}{G^{\pi}}$

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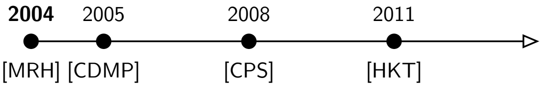
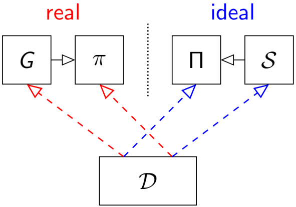
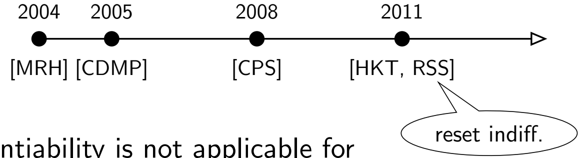


- composition theorem by Maurer, Renner, and Holenstein [MRH04]
- proof in Π model $\rightsquigarrow$ proof in π model, given indiff. construction G^{π}



- e.g., G: constructed "random oracle"; π: ideal cipher; Π: real random oracle
- ask for simulator $\mathcal{S}$ such that $(G^{\pi}, \pi) \overset{c}{\approx} (\Pi, \mathcal{S}^{\Pi})$

Limitations of Indifferentiability



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- $\ldots, x \leftarrow \mathcal{A}_1, \ldots, y \leftarrow \mathcal{A}_2, \ldots$

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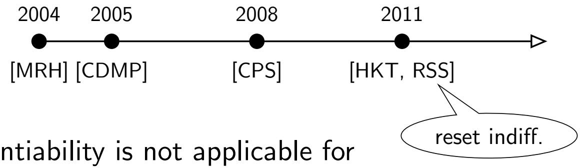

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no shared state

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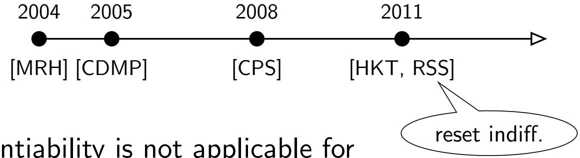

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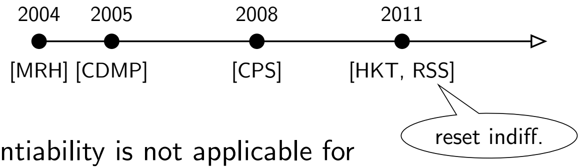

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- problem (roughly): distinct stages result in distinct simulators, distinct simulators are inconsistent
- allow the distinguisher to reset the simulator, reset indifferentiability [RSS11]

$$\text{ROM} \stackrel{?}{\equiv} \text{ICM}$$

ROM $\stackrel{?}{\equiv}$ ICM, Revisited

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- constructions in [CDMP05, CPS08, HKT11] are not reset indifferentiable
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- reset-indifferentiable constructions cannot be domain extending [LAMP12, DGHM13]
  - assuming that ROs have infinite domain, ICM $\not\Rightarrow$ ROM

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1. under reset indifferentiability, ROM $\not\equiv$ ICM
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2. "Duality Lemma": two primitives are either equivalent or incomparable
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   - (no result for length-preserving constructions)
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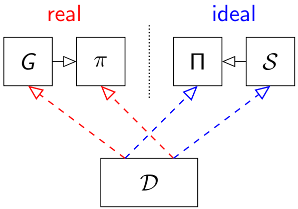
Multi-Stage Indifferentiability



- instead of resettable simulators, consider stateless ones
- think "reset after each query"
- equivalent to reset indifferentiability
- simulators are *pseudo deterministic*—why?

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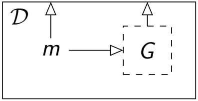

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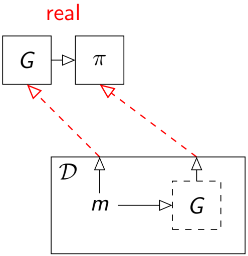

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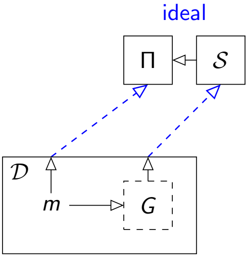

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- ideal world
  - $\mathcal{S}$ needs to query Π on m
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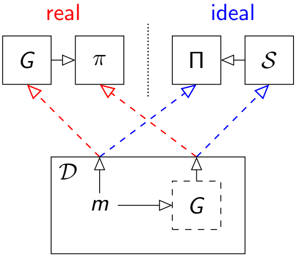

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- note: choice of primitives arbitrary

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- can switch roles!

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- positive (resp. negative) result in one direction translates to other direction
- no domain-extending constructions $\Rightarrow$ no domain-shrinking constructions; ROM and ICM are incomparable

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- turns out n-reset $=$ n'-reset $=$ 1-reset
- idea: at least one reset must be "critical", find it

Eliminating Resets

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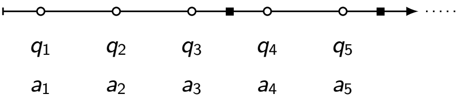

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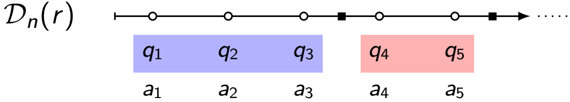

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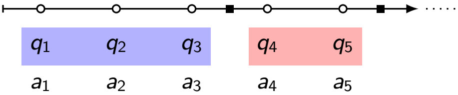

construct $\mathcal{D}_1$

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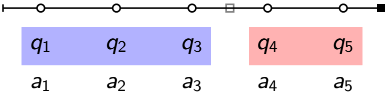

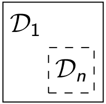

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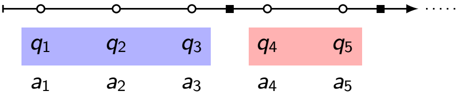

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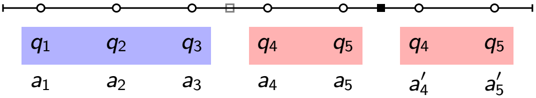

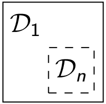

let $\mathcal{D}_1$'s output be $(a_4, a_5) \stackrel{?}{=} (a'_4, a'_5)$

next, consider $\mathcal{D}_{n-1}$

Summary

take-home message

- is the ROM equivalent to the ICM?
- answer—depends on "equivalent"
  - for composing single-stage games: ✓
  - multi stage / non length preserving: ✗
  - multi stage / length preserving:

open question

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The End

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References I

Mihir Bellare and Phillip Rogaway.
Random oracles are practical: A paradigm for designing efficient protocols.
In V. Ashby, editor, *ACM CCS 93*, pages 62–73. ACM Press, November 1993.

Jean-Sébastien Coron, Yevgeniy Dodis, Cécile Malinaud, and Prashant Puniya.
Merkle-Damgård revisited: How to construct a hash function.
In Victor Shoup, editor, *CRYPTO 2005*, volume 3621 of *LNCS*, pages 430–448. Springer, August 2005.

Jean-Sébastien Coron, Jacques Patarin, and Yannick Seurin.
The random oracle model and the ideal cipher model are equivalent.
In David Wagner, editor, *CRYPTO 2008*, volume 5157 of *LNCS*, pages 1–20. Springer, August 2008.

Gregory Demay, Peter Gazi, Martin Hirt, and Ueli Maurer.
Resource-restricted indifferentiability.
In Thomas Johansson and Phong Q. Nguyen, editors, *EUROCRYPT 2013*, volume 7881 of *LNCS*, pages 664–683. Springer, May 2013.

Thomas Holenstein, Robin Künzler, and Stefano Tessaro.
The equivalence of the random oracle model and the ideal cipher model, revisited.
In Lance Fortnow and Salil P. Vadhan, editors, *43rd ACM STOC*, pages 89–98. ACM Press, June 2011.

Atul Luykx, Elena Andreeva, Bart Mennink, and Bart Preneel.
Impossibility results for indifferentiability with resets.
Cryptology ePrint Archive, Report 2012/644, 2012.
`http://eprint.iacr.org/`.

Ueli M. Maurer, Renato Renner, and Clemens Holenstein.
Indifferentiability, impossibility results on reductions, and applications to the random oracle methodology.
In Moni Naor, editor, *TCC 2004*, volume 2951 of *LNCS*, pages 21–39. Springer, February 2004.

References II

Thomas Ristenpart, Hovav Shacham, and Thomas Shrimpton.
Careful with composition: Limitations of the indifferentiability framework.
In Kenneth G. Paterson, editor, *EUROCRYPT 2011*, volume 6632 of *LNCS*, pages 487–506. Springer, May 2011.

Claude E. Shannon.
Communication theory of secrecy systems.
Bell Systems Technical Journal, 28(4):656–715, 1949.